

## Pre-Calculus Final I Part II Review Sheet

Do all work on loose leaf.

Factor each of the following.

1.  $3x^5 - 6x^3 + 12x^2$

2.  $6x^5y^2 + 9x^2y^3$

3.  $x^2 + 4x - 21$

4.  $x^2 + 8x + 15$

5.  $3x^2 - 12x - 15$

6.  $9x^2 - 100$

7.  $8x^2 - x - 7$

8.  $3a^3 - 15a^2 + 2a - 10$

9.  $x^3 - 64$

10.  $8x^3 + 27$

Perform the indicated operation.

11.  $\frac{3x^2}{3-x} \cdot \frac{x^2 - 7x + 12}{6x^5}$

12.  $\frac{x^2 - 12x + 27}{x^2 - 81} \cdot \frac{x+9}{3-x}$

13.  $\frac{x^2 - 4}{2y} \div \frac{x^2 - 2x}{xy}$

Find and simplify the difference quotient given each function below.  $\left(DQ = \frac{f(x+h) - f(x)}{h}\right)$

14.  $f(x) = 5x$

15.  $f(x) = 8x - 11$

16.  $f(x) = x^2 - 13x + 5$

Put each of the equations in standard form of a circle. Then, identify the center and radius.

17.  $x^2 + y^2 - 2x + 4y + 3 = 0$

18.  $x^2 + y^2 + 10x - 6y + 12 = 0$

Simplify each of the following.

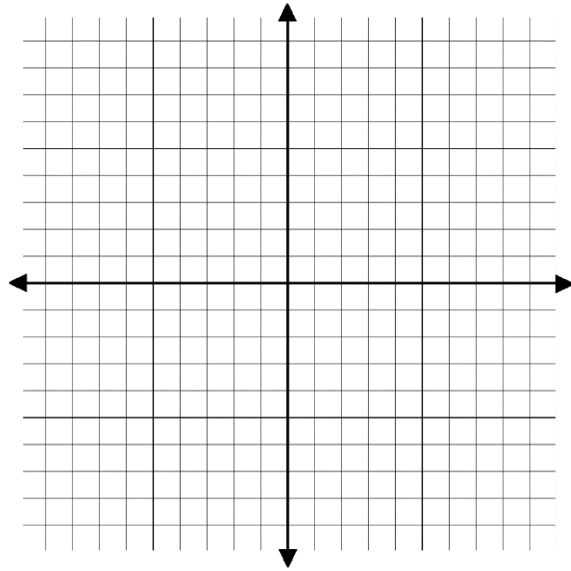
19.  $\frac{1}{2+4i}$

20.  $\frac{5-3i}{1-i}$

21.  $\frac{3-5i}{2i}$

Graph the following using the 4 key features. (opens up/down, vertex, x-intercepts, y-intercepts)

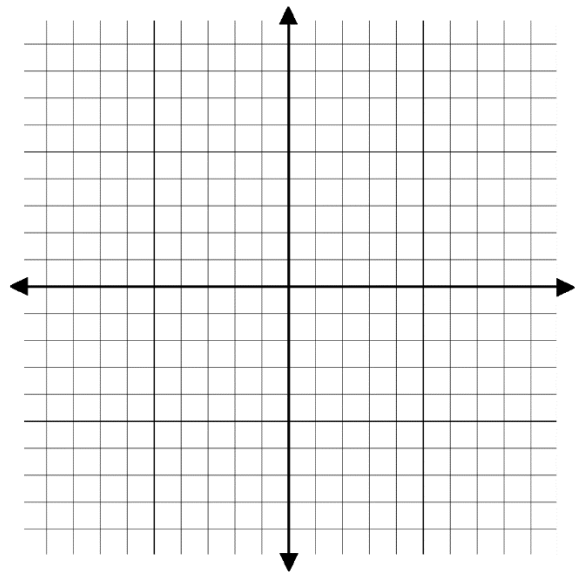
22.  $y = (x + 4)^2 - 9$



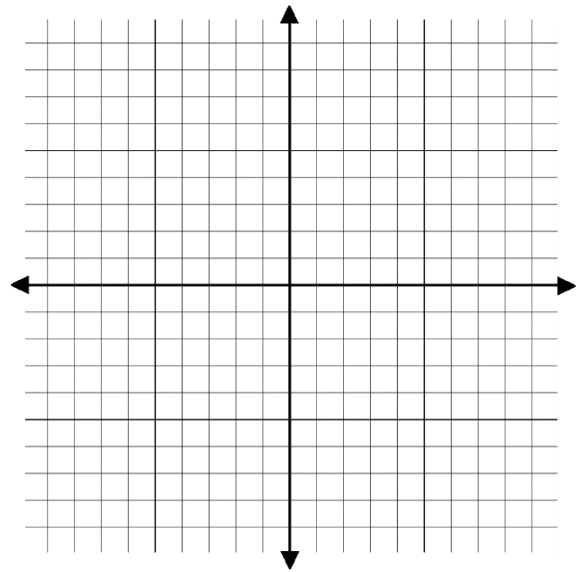
23. A rock is thrown vertically from the ground with a velocity of 24 meters per second and it reaches a height of  $2 + 24t - 4.9t^2$  after  $t$  seconds. How many seconds after the rock is thrown will it reach maximum height, and what is the maximum height the rock will reach, in meters? Round your answers to the nearest hundredth.

Graph each function. (end behavior, x-intercepts, including multiplicities and descriptions, y-intercept, and maximum number of turning points)

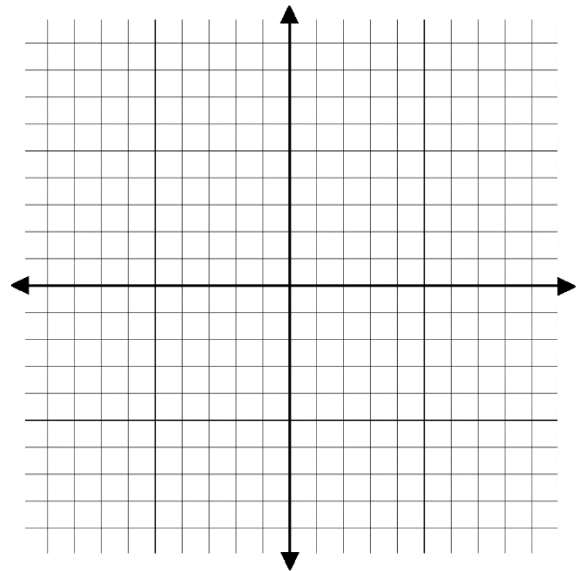
24.  $f(x) = x^3 - x^2 - 9x + 9$



25.  $f(x) = 4x - x^3$



26.  $f(x) = -x^4 + 6x^3 - 9x^2$



Divide using long division.

27.  $(10x^3 - 26x^2 + 17x - 13) \div (5x - 3)$

28.  $\frac{3x^4 - 2x^2 - 10x}{x - 2}$

Solve each of the following using the possible zeros and synthetic division to solve the remaining equation.

29.  $f(x) = x^3 - 2x^2 - 11x + 12$

30.  $2x^3 - 5x^2 - 6x + 4 = 0$

31. Create the equation of the cubic, in standard form, that has x-intercepts of -6, 4, and 2 and passes through the point (3,-18).

Solve each equation.

32.  $\log_2(x^2 - 14x) = 5$

33.  $4\log_5 x - \log_5 4 = \log_5 4$

34.  $\log_4(x+3) + \log_4(x-3) = 2$

35.  $\log_5(x+3) - \log_5 x = \log_5 4$

36.  $5 + 2\ln x = 4$

37.  $6 \ln 4x - 1 = 14$

38. In 2005, your great-aunt Marguerite retired after forty years of service to a Wall Street firm and received a retirement bonus of \$500,000. Because of the large sum, she was able to invest it at an annual rate of 7.25% compounded continuously. Use the formula  $A = Pe^{rt}$  to determine in how many years, to the nearest tenth of a year, will your great-aunt's money have doubled?

39. A liquid with an initial temperature of 194 °F cools in a room whose temperature is held at 68 °F. The temperature of the liquid,  $T$ , as it cools can be modeled as a function of time,  $x$ , using:

$$T(x) = (T_i - T_r)e^{kx} + T_r$$

Where  $T_i$  is the initial temperature,  $T_r$  is the temperature of the room and  $k$  is the decay constant.

- (a) If  $T(15) = 102$  then find the value of  $k$  accurate to the nearest hundredth.

- (b) How many minutes does the model predict it will take for the liquid to reach a temperature of 70 °F? Round to the nearest minute and show or explain how you arrived at your answer.

ANSWER KEY

1.  $3x^2(x^2 - 2x + 4)$

2.  $3x^2y^2(2x^3 + 3y)$

3.  $(x + 7)(x - 3)$

4.  $(x + 3)(x + 5)$

5.  $3(x - 5)(x + 1)$

6.  $(3x - 10)(3x + 10)$

7.  $(8x + 7)(x - 1)$

8.  $(3a^2 + 2)(a - 5)$

9.  $(x - 4)(x^2 + 4x + 16)$

10.  $(2x + 3)(4x^2 - 6x + 9)$

11.  $\frac{-(x-4)}{2x^3}$

12. -1

13.  $\frac{x+2}{2}$

14. 5

15. 8

16.  $2x + h - 13$

17.  $(x - 1)^2 + (y + 2)^2 = 2$  center: (1,-2)  $r = \sqrt{2}$

18.  $(x + 5)^2 + (y - 3)^2 = 22$  center: (-5,3)  $r = \sqrt{22}$

19.  $\frac{1}{10} - \frac{i}{5}$

20.  $4 + i$

21.  $\frac{-5}{2} - \frac{3i}{2}$

22. opens up

23. 2.4 sec, 31.4 feet

vertex: (-4,-9)

x-int: {-7,-1}

y-int: {7}

24. falls left, rises right

$x = -3, 3, 1$

$f(x) = 9$

$n - 1 = 2$

25. rises left, falls right

$x = -2, 0, 2$

$f(x) = 0$

$n - 1 = 2$

26. falls left and right

$x = 0, 3$

$f(x) = 0$

$n - 1 = 3$

27.  $2x^2 - 4x + 1 - \frac{10}{5x-3}$

28.  $3x^3 + 6x^2 + 10x + 10 + \frac{20}{x-2}$

29. {-3,1,4}

30.  $\left\{\frac{1}{2}, 1 \pm \sqrt{5}\right\}$

31.  $a = 2$       $f(x) = 2x^3 - 56x + 96$

32.  $x = -2$  or  $x = 16$

33.  $x = 2$

34.  $x = 5$

35.  $x = 1$

36.  $x = .61$

37.  $x = 3.05$

38.  $t = 9.6$  years

39. a)

$T(x) = (194 - 68)e^{kx} + 68$ $T(x) = 126e^{kx} + 68$	$\Rightarrow$	$102 = 126e^{k-15} + 68$ $102 = 126e^{15k} + 68$ $34 = 126e^{15k}$ $e^{15k} = \frac{34}{126}$	$\Rightarrow$	$\ln(e^{15k}) = \ln\left(\frac{34}{126}\right)$ $15k = \ln\left(\frac{34}{126}\right)$ $k = \frac{\ln\left(\frac{34}{126}\right)}{15} = -.08732 = -.09$
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(b)

$T(x) = 126e^{-.09x} + 68$ $70 = 126e^{-.09x} + 68$ $2 = 126e^{-.09x}$ $e^{-.09x} = \frac{2}{126}$	$\Rightarrow$	$\ln(e^{-.09x}) = \ln\left(\frac{2}{126}\right)$ $-.09x = \ln\left(\frac{2}{126}\right)$ $x = \frac{\ln\left(\frac{2}{126}\right)}{-.09} = 46 \text{ min}$
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